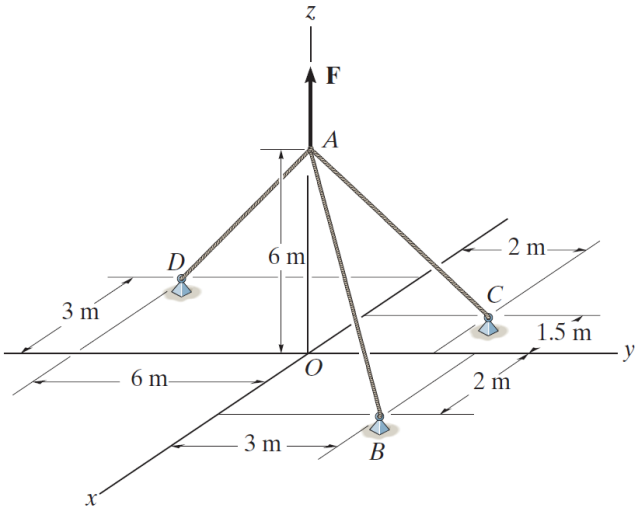
Problem R3-8

If cable AB is subjected to a tension of 700 N, determine the tension in cables AC and AD and the magnitude of the vertical force **F**.



Prob. R3–8

Solution

Write position vectors to points A, B, C and D.

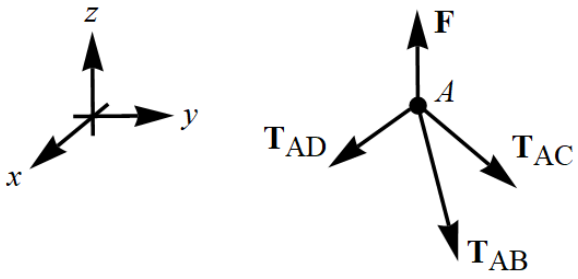
$$\mathbf{r}_A = \langle 0, 0, 6 \rangle \text{ m}$$

$$\mathbf{r}_B = \langle 2, 3, 0 \rangle \text{ m}$$

$$\mathbf{r}_C = \langle -1.5, 2, 0 \rangle \text{ m}$$

$$\mathbf{r}_D = \langle -3, -6, 0 \rangle \text{ m}$$

Draw a free-body diagram for the knot at A.



In order for the system to be in equilibrium, the sum of the forces must be zero.

$$\mathbf{T}_{\text{AB}} + \mathbf{T}_{\text{AC}} + \mathbf{T}_{\text{AD}} + \mathbf{F} = \mathbf{0}$$

$$T_{\text{AB}}\hat{\mathbf{u}}_{\text{AB}} + T_{\text{AC}}\hat{\mathbf{u}}_{\text{AC}} + T_{\text{AD}}\hat{\mathbf{u}}_{\text{AD}} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

$$T_{\text{AB}}\frac{\mathbf{r}_B - \mathbf{r}_A}{|\mathbf{r}_B - \mathbf{r}_A|} + T_{\text{AC}}\frac{\mathbf{r}_C - \mathbf{r}_A}{|\mathbf{r}_C - \mathbf{r}_A|} + T_{\text{AD}}\frac{\mathbf{r}_D - \mathbf{r}_A}{|\mathbf{r}_D - \mathbf{r}_A|} + F\langle 0, 0, 1 \rangle = \mathbf{0}$$

Write out the unit vectors and simplify the left side.

$$T_{\text{AB}}\frac{\langle 2-0,3-0,0-6\rangle}{\sqrt{(2-0)^2+(3-0)^2+(0-6)^2}}+T_{\text{AC}}\frac{\langle -1.5-0,2-0,0-6\rangle}{\sqrt{(-1.5-0)^2+(2-0)^2+(0-6)^2}}$$
$$+T_{\text{AD}}\frac{\langle -3-0,-6-0,0-6\rangle}{\sqrt{(-3-0)^2+(-6-0)^2+(0-6)^2}}+F\langle 0,0,1\rangle=\mathbf{0}$$

$$T_{\text{AB}}\left\langle\frac{2}{7},\frac{3}{7},-\frac{6}{7}\right\rangle+T_{\text{AC}}\left\langle-\frac{3}{13},\frac{4}{13},-\frac{12}{13}\right\rangle+T_{\text{AD}}\left\langle-\frac{1}{3},-\frac{2}{3},-\frac{2}{3}\right\rangle+F\langle 0,0,1\rangle=\mathbf{0}$$

$$\left\langle\frac{2}{7}T_{\text{AB}}-\frac{3}{13}T_{\text{AC}}-\frac{1}{3}T_{\text{AD}},\ \frac{3}{7}T_{\text{AB}}+\frac{4}{13}T_{\text{AC}}-\frac{2}{3}T_{\text{AD}},\ -\frac{6}{7}T_{\text{AB}}-\frac{12}{13}T_{\text{AC}}-\frac{2}{3}T_{\text{AD}}+F\right\rangle=\langle 0,0,0\rangle$$

Match the components to get a system of equations.

$$\left.\begin{aligned}\frac{2}{7}T_{\text{AB}}-\frac{3}{13}T_{\text{AC}}-\frac{1}{3}T_{\text{AD}}&=0\\ \frac{3}{7}T_{\text{AB}}+\frac{4}{13}T_{\text{AC}}-\frac{2}{3}T_{\text{AD}}&=0\\ -\frac{6}{7}T_{\text{AB}}-\frac{12}{13}T_{\text{AC}}-\frac{2}{3}T_{\text{AD}}+F&=0\end{aligned}\right\}$$

Solve it for T_{AC}, T_{AD}, and F and then plug in $T_{\text{AB}}=700$ N.

$$T_{\text{AC}}=\frac{13}{70}T_{\text{AB}}=130.\text{ N}$$
$$T_{\text{AD}}=\frac{51}{70}T_{\text{AB}}=510.\text{ N}$$
$$F=\frac{53}{35}T_{\text{AB}}=1.06\times 10^3\text{ N}$$